

Conformal sequestering simplified

Martin Schmaltz

*Physics Department, Boston University,
590 Commonwealth Ave, Boston, MA 02215, U.S.A.
E-mail: schmaltz@bu.edu*

Raman Sundrum

*Department of Physics and Astronomy, Johns Hopkins University,
3400 North Charles St., Baltimore, MD 21218, U.S.A.
E-mail: sundrum@pha.jhu.edu*

ABSTRACT: Sequestering is important for obtaining flavor-universal soft masses in models where supersymmetry breaking is mediated at high scales. We construct a simple and robust class of hidden sector models which sequester themselves from the visible sector due to strong and conformally invariant hidden dynamics. Masses for hidden matter eventually break the conformal symmetry and lead to supersymmetry breaking by the mechanism recently discovered by Intriligator, Seiberg and Shih. We give a unified treatment of subtleties due to global symmetries of the CFT. There is enough review for the paper to constitute a self-contained account of conformal sequestering.

KEYWORDS: Supersymmetry and Duality, Supersymmetry Breaking, Supersymmetry Phenomenology.

Contents

1.	Introduction	1
2.	Toy models of conformal sequestering	4
2.1	Non-supersymmetric example	4
2.2	Supersymmetric QCD	6
3.	A Self-sequestering hidden sector	7
4.	Sequestering in detail	10
4.1	Relevant hidden superpotential couplings	11
4.2	Marginal flavor-symmetry “currents”	12
4.3	Safe and unsafe currents	14
4.4	Gauging flavor to suppress harmful non-abelian currents	15
4.5	Strong superpotential coupling to suppress harmful abelian current	15
4.6	Emergent symmetries?	15
4.7	Integrating out visible F_Φ -terms	17
5.	Discussion and generalizations	17
A.	SUSY QCD Sequestering	20

1. Introduction

Several of the effective field theory solutions to the flavor problem of weak scale SUSY rely on “sequestering” of the hidden sector from the visible sector, in order to avoid flavor-violating squark and slepton masses from operators of the form

$$\int d^4\theta c_{ij} \frac{\Phi_i^\dagger \Phi_j X^\dagger X}{M_{Pl}^2}. \tag{1.1}$$

Here, Φ_i stands for the i 'th generation of an MSSM quark or lepton superfield, X is a hidden sector superfield with a SUSY breaking F_X -component and M_{Pl} is the Planck scale. This operator arises from integrating out heavy (string) physics near the Planck scale and is expected to have coefficients, $c_{ij} \sim \mathcal{O}(1)$, which are flavor-violating because Yukawa couplings require breaking of flavor symmetries.

In order for the flavor-preserving scalar masses generated from anomaly-mediated SUSY breaking (AMSB) [1, 2], gaugino-mediation [3, 4], or any other such high-scale mediation mechanism [5–10] to dominate, the operator of eq. (1.1) must be suppressed.

For example, the (flavor-universal) contributions to visible scalar masses from AMSB are loop-factor suppressed relative to those from the above direct coupling to the hidden sector,

$$m_{AMSB}^2 \sim \left(\frac{g_{SM}^2}{16\pi^2} \right)^2 \frac{|F_X|^2}{M_{Pl}^2} \quad (1.2)$$

$$(m_{\text{direct}}^2)_{ij} = c_{ij} \frac{|F_X|^2}{M_{Pl}^2} . \quad (1.3)$$

This implies that eq. (1.1) must be suppressed by at least $\mathcal{O}(10^{-6} - 10^{-7})$ for AMSB to solve the flavor problem. Sequestering refers to this suppression, even beyond Planck-suppression, of direct hidden-visible couplings.

Conformal sequestering [11, 12] accomplishes this suppression by strong-coupling hidden sector anomalous dimensions (or alternatively by large visible-sector anomalous dimensions [13, 14]) in the running of dangerous operators such as eq. (1.1) from the Planck scale down to the SUSY-breaking intermediate scale,

$$\Lambda_{\text{int}} \equiv \sqrt{F_X} \sim 10^{11} \text{ GeV} . \quad (1.4)$$

A virtue of conformal sequestering is that it depends on purely four-dimensional, renormalizable dynamics, rather than non-renormalizable extra-dimensional effective field theories as originally proposed [1]. The robustness and plausibility of sequestering can therefore be addressed with less questionable assumptions about string theory ultraviolet completions [15, 16]. This is still a non-trivial task because of the key role played by non-perturbative strong dynamics, and progress depends on inferences based on global symmetries, SUSY and field theory dualities. The central obstacle for conformally sequestered models of SUSY breaking is that the symmetries used to understand strongly-coupled SUSY dynamics also yield conserved currents with vanishing, rather than the requisite large, anomalous dimensions [11–14]. Nevertheless successful models have been constructed. While early models were somewhat complicated [11, 12], technology has improved and more plausible models with conformal sequestering now exist [17, 18].

In this paper, we present a particularly simple class of hidden sector models which achieve conformal sequestering suitable for AMSB by taking advantage of the SUSY breaking metastable vacua of supersymmetric QCD recently discovered by Intriligator, Seiberg and Shih (ISS) [19]. The simplicity of our models paints a highly plausible picture of how anomaly-mediation¹ might dominate weak scale SUSY breaking. We also provide a broader perspective and collect general results on model building. Most of these have appeared previously in the literature [11–14, 17, 18, 21–24], but the arguments of sections 4.2–4.3 on “exact flavor currents” and section 4.6. on emergent symmetries are new. We have tried to incorporate enough review material to make this paper a self-contained introduction to conformal sequestering.

¹In this paper we are using AMSB as an example for a flavor-blind high scale mediation mechanism. Of course, fully realistic mediation mechanisms (with positive slepton masses) require additional structure [20], but our focus here is on the issues of conformal sequestering and hidden sector SUSY breaking.

Here is the basic plot of conformal sequestering. Hidden sector dynamics is assumed to be in the vicinity of a strongly-coupled superconformal fixed point over a large enough hierarchy between the M_{Pl} and Λ_{int} so that strong running effects can suppress dangerous hidden-visible couplings. We focus on the resulting visible *scalar* mass-squareds because other soft terms can be protected by chiral symmetries. We work in flat spacetime even though AMSB relies on supergravity since we are only addressing the issue of sequestering, the suppression of the unwanted “background” to AMSB. Since at strong coupling it is not obvious which operators are the most relevant and therefore most dangerous, we discuss sequestering of a general hidden sector operator \mathcal{O}_{hid} . Denoting the scalar component of Φ by ϕ and, for simplicity, working with the component Lagrangian, we have

$$\begin{aligned}\Delta\mathcal{L}_{\text{hid-vis}}(M_{Pl}) &\sim \frac{1}{M_{Pl}^{n-2}} \phi^\dagger\phi \mathcal{O}_{\text{hid}} \\ \Delta\mathcal{L}_{\text{hid-vis}}(\mu) &\sim \left(\frac{\mu}{M_{Pl}}\right)^\gamma \frac{1}{M_{Pl}^{n-2}} \phi^\dagger\phi \mathcal{O}_{\text{hid}},\end{aligned}\tag{1.5}$$

where n and γ are the canonical and anomalous dimensions of \mathcal{O}_{hid} respectively. Here we neglect the presumably weak visible running effects. If this strong running holds down to energies not far above Λ_{int} , then (except for the AMSB contribution),

$$m_\phi^2 \sim \frac{\Lambda_{\text{int}}^4}{M_{Pl}^2} \left(\frac{\Lambda_{\text{int}}}{M_{Pl}}\right)^{n+\gamma-4},\tag{1.6}$$

which is a suppression of the direct contribution, eq. (1.3), by an additional $(\frac{\Lambda_{\text{int}}}{M_{Pl}})^{n+\gamma-4}$. This is sufficient for solving the flavor problem provided

$$n + \gamma - 4 \gtrsim 1\tag{1.7}$$

for all hidden sector operators \mathcal{O}_{hid} .

What kind of fixed point satisfies this condition for all Lorentz-invariant operators? When asking about visible masses, it is sufficient to think of $\phi^\dagger\phi$ at zero momentum, that is as a spacetime constant. In this case, as far as the hidden dynamics in the conformal regime is concerned, eq. (1.5) is just adding the interaction \mathcal{O}_{hid} to the fixed point with a small ϕ -dependent coupling. If the fixed point is IR-attractive, then all such perturbations should be irrelevant, so that the scaling dimension of \mathcal{O}_{hid} , $n + \gamma$, is larger than 4. At a strongly-coupled fixed point with no small numbers, irrelevant couplings are $\mathcal{O}(1)$ irrelevant, which is the condition, eq. (1.7). Thus, we must simply arrange for the hidden sector dynamics to be close to a strongly-coupled IR-attractive fixed-point below the Planck scale in order to sequester. However, as mentioned above, there is an important subtlety involving global symmetries which we treat carefully in this paper.

Ultimately the hidden sector must break SUSY at Λ_{int} , and thus must violate conformal symmetry above this scale. We introduce hidden masses $m_X \gtrsim \Lambda_{\text{int}}$, below which the hidden dynamics triggers spontaneous SUSY breaking. The relevant scales are summarized in figure 1.

This paper is organized as follows. In section 2, we give a quick introduction to conformal sequestering within two toy models, without reference to SUSY breaking. The

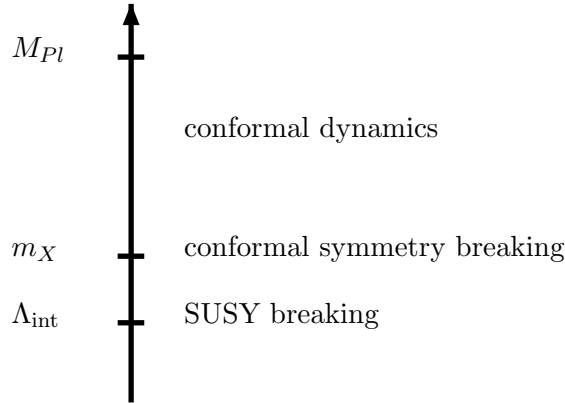


Figure 1: Scales and dynamics.

first toy model is non-supersymmetric, giving the simplest concrete illustration of conformal sequestering. The second toy model is just SUSY QCD, and we give a new streamlined discussion of conformal sequestering there, with some back-up material reviewed in an appendix. In section 3, we introduce our new class of strongly coupled hidden sector models, explain how the UV conformal regime gives way to SUSY breaking in the IR, and briefly indicate how conformal sequestering is accomplished as far as terms of the form of eq. (1.1) are concerned. In section 4, we carefully examine the various aspects of sequestering in our new class of models, including subtleties previously neglected. In section 5, we broaden the considerations of sections 3 and 4 to a set of model-building rules, both for the purpose of constructing new models as well as for assessing the general plausibility of sequestering and the high-scale mediation mechanisms that depend on it.

The casual reader, wanting a quick acquaintance with our new models and a summary of how they perform, need only read the Introduction and section 3.

2. Toy models of conformal sequestering

In this section we use two simple examples to demonstrate conformal sequestering. The first is a scalar theory with a quartic coupling in $4 - \epsilon$ dimensions. It has a non-trivial infrared (IR) attractive fixed point at weak coupling which allows us to give explicit formulae in perturbation theory. The theory is also non-supersymmetric, which makes it clear that conformal sequestering is a property of conformal theories, not just SUSY. Our second example is supersymmetric QCD in four dimensions in the conformal window.

2.1 Non-supersymmetric example

Consider a single massless real scalar field \hat{X} with a quartic interaction in $4 - \epsilon$ dimensions

$$\mathcal{L} = \frac{1}{2} \left(\partial_\mu \hat{X} \right)^2 - \mu^\epsilon \frac{\lambda}{4!} \hat{X}^4 . \quad (2.1)$$

We gave the field \hat{X} a hat as a reminder that \hat{X} has a canonically normalized kinetic term and we factored out the explicit factor of μ^ϵ from the coupling constant to make λ dimensionless. With μ^ϵ factored out, the Lagrangian is invariant under conformal transformations iff λ is scale invariant (μ independent).

The $\overline{\text{MS}}$ beta function for λ at one loop is

$$\beta(\lambda) = \mu \frac{\partial \lambda}{\partial \mu} - \epsilon \lambda + 3 \frac{\lambda^2}{16\pi^2} \quad (2.2)$$

The first term is the “classical scaling”, it comes from the explicit factor of μ^ϵ in the interaction in $4 - \epsilon$ dimensions. The second term is the one-loop correction. The beta function vanishes for $\lambda = \lambda_* = \frac{16\pi^2 \epsilon}{3}$ and the theory is conformal. To study the theory near conformality we expand the beta function near the fixed point to linear order

$$\beta(\lambda) \simeq 0 + \left. \frac{\partial \beta}{\partial \lambda} \right|_{\lambda=\lambda_*} (\lambda - \lambda_*) = \epsilon (\lambda - \lambda_*) \quad (2.3)$$

and integrate to obtain

$$\lambda(\mu) = \lambda_* + \left(\frac{\mu}{M} \right)^\epsilon (\lambda(M) - \lambda_*) \quad (2.4)$$

We see that the fixed point is attractive and that deviations from the fixed point are scaled away by a factor of $(\mu/M)^\epsilon$, that is irrelevant by ϵ , despite the fact that the coupling is canonically “relevant” by ϵ .

We used perturbation theory to compute the beta function and establish the existence of a fixed point. At strong coupling we cannot prove the existence of the fixed point, but if we assume that an isolated IR fixed point exists, we can expand the beta function near the fixed point and derive power law running, corresponding to deviations from the fixed point being irrelevant by $\partial\beta/\partial\lambda|_{\lambda=\lambda_*} \equiv \beta'_*$.

To set the stage for what comes later, let us translate these results into a different basis reached by rescaling the field \hat{X} to make the coupling constant equal to λ_* at all energy scales (λ_* is defined as the fixed point coupling in the canonical basis). The kinetic term is now multiplied by a Z factor

$$\mathcal{L} = \frac{1}{2} Z(\mu) (\partial_\mu X)^2 - \mu^\epsilon \frac{\lambda_*}{4!} X^4, \quad (2.5)$$

which accounts for all non-trivial running. We determine $Z(\mu)$ from (2.4) by rescaling $X = (\lambda(\mu)/\lambda_*)^{1/4} \hat{X}$ to obtain

$$Z(\mu) = \sqrt{\frac{\lambda_*}{\lambda}} = \left[\frac{1}{1 + \left(\frac{\mu}{M}\right)^\epsilon \left(\frac{\lambda(M)}{\lambda_*} - 1\right)} \right]^{1/2} \quad (2.6)$$

$$= \left[\frac{1}{1 + \left(\frac{\mu}{M}\right)^\epsilon (Z_M^{-2} - 1)} \right]^{1/2} \simeq 1 + \left(\frac{\mu}{M}\right)^\epsilon (Z_M - 1) \quad (2.7)$$

where in the second line we traded the UV boundary value of the coupling $\lambda(M)$ for the corresponding UV value of the wave function $Z_M \equiv \sqrt{\lambda_*/\lambda(M)}$, and approximated for Z_M close to 1. In this basis, the approach to the fixed point is $Z(\mu) \rightarrow 1$ when $\mu \rightarrow 0$; deviations from the fixed point are again seen to be irrelevant by ϵ .

Let us now discuss sequestering by imagining that our toy model is the “hidden sector” coupled to a “visible sector” represented by a free real scalar field ϕ . We assume that the higher dimensional operator

$$c \frac{\phi^2}{M^2} \frac{1}{2} (\partial_\mu X)^2 \tag{2.8}$$

couple the two sectors at the UV scale M . Note that there is no hat on the X because we are using the basis discussed in the previous paragraphs.

We now show that this operator sequesters, i.e. it obtains an anomalous dimension from hidden sector dynamics which increases its scaling dimension so that it becomes more irrelevant in the IR than the naive $1/M^2$. One can compute the anomalous dimension directly by computing Feynman diagrams in which the operator is dressed with X loops but it is easier to obtain the result with a trick [25].

The argument uses the fact that the visible sector fields are background fields in this calculation (the running of the operator due to visible sector interactions is negligible compared to hidden sector running), and we may choose a simple configuration for ϕ . The most convenient choice is a constant $\phi = \phi_0 \ll M$. Then, from the point of view of the hidden sector dynamics, the operator (2.8) becomes a small additional contribution to the kinetic term for X . More precisely, Z_M gets a contribution equal to $c \phi_0^2/M^2$. But we know how the X kinetic term is renormalized from equation (2.7), any dependence on Z_M and therefore any dependence on $c \phi_0^2/M^2$ scales away with a factor of $(\mu/M)^\epsilon$. This scaling is valid for any constant ϕ and therefore also for the operator

$$\left(\frac{\mu}{M}\right)^\epsilon c \frac{\phi^2}{M^2} \frac{1}{2} (\partial_\mu X)^2 . \tag{2.9}$$

The essence of the argument is that for constant ϕ the operator (2.8) can be thought of as a contribution to the UV boundary conditions for a coupling in the hidden sector theory. But since the hidden sector runs towards an IR fixed point, the theory “forgets” the UV values of the coupling constants. In the IR, the coupling constants depend only on the CFT, not on the UV boundary conditions. Numerically, the “forgetting” proceeds by power law scaling. Therefore the operator (2.8) must scale away with a power of μ/M . This argument does not depend on perturbation theory, it relies only on the assumption that we have an IR attractive fixed point. In the strongly coupled case the exponent would be β'_* . It should also be clear that the argument generalizes to CFTs with multiple couplings. Again, UV perturbations to an IR attractive fixed point, due to the visible sector, simply scale away.

2.2 Supersymmetric QCD

While running Z -factors are unusual in a discussion of simple scalar field theories they are very natural in supersymmetric theories. In our second example [11, 12], supersymmetric QCD in the conformal window, all running except for a one-loop contribution to gauge couplings occurs in Z -factors when working in a holomorphic basis. As in the example discussed above, it is most convenient to work in a basis where all running (including

the one-loop running of the holomorphic gauge coupling) has been scaled into Z -factors for the matter fields. This is accomplished by performing a “Konishi-anomalous” [26] transformation of the quark fields. The Lagrangian for SUSY QCD is then

$$\mathcal{L} = R(\mu) \int d^4\theta (Q^\dagger Q + \bar{Q}^\dagger \bar{Q}) + \int d^2\theta W_\alpha W^\alpha + h.c. \quad (2.10)$$

where $R(\mu)$ is the running Z -factor. Near the fixed point ($R \rightarrow R_* = \text{const.}$) one finds (see [11] and the appendix)

$$R(\mu)/R_* = [R(M)]^{(\frac{\mu}{M})^{\beta'_*}} \simeq 1 + \left(\frac{\mu}{M}\right)^{\beta'_*} (R(M) - 1) \quad (2.11)$$

As in the previous example, deviations from the fixed point scale away with a power of μ/M . β'_* is the derivative of the β function for the gauge coupling at the fixed point, it is not calculable at strong coupling but is expected to be of order 1.

The operator we wish to sequester with conformal dynamics is of the form

$$\int d^4\theta c \frac{\Phi^\dagger \Phi}{M^2} (Q^\dagger Q + \bar{Q}^\dagger \bar{Q}) . \quad (2.12)$$

Again, by using the trick of considering constant (scalar) background values for the field Φ so that $c \frac{\Phi^\dagger \Phi}{M^2}$ becomes a number which can be absorbed into $R(M)$, we can derive the sequestering from the scaling (2.11),

$$\int d^4\theta \left(\frac{\mu}{M}\right)^\gamma c \frac{\Phi^\dagger \Phi}{M^2} (Q^\dagger Q + \bar{Q}^\dagger \bar{Q}), \quad (2.13)$$

where the anomalous dimension associated to small Z -shifts (the kinetic operator) is $\gamma = \beta'_*$.

3. A Self-sequestering hidden sector

In this section we present a hidden sector model which exhibits conformal sequestering and spontaneous SUSY breaking. We use this model as a concrete example because of its simplicity. Similar models are straightforward to construct.

The model is $\mathcal{N}=1$ supersymmetric, with gauge group $SU(N)$ and F chiral superfield “flavors” Q, \bar{Q} in the fundamental and anti-fundamental representations and an adjoint A .

	SU(N)	SU(F)	SU(F)	
Q	\square	\square	1	(3.1)
\bar{Q}	$\bar{\square}$	1	$\bar{\square}$	
A	<i>adj</i>	1	1	

For the range of flavors of interest, $N < F < \frac{3}{2}N$, the $SU(N)$ gauge coupling is asymptotically free and grows in the infrared. In order to reduce the number of allowed couplings we weakly gauge the diagonal “vector” $SU(F)$ group. We choose this gauge

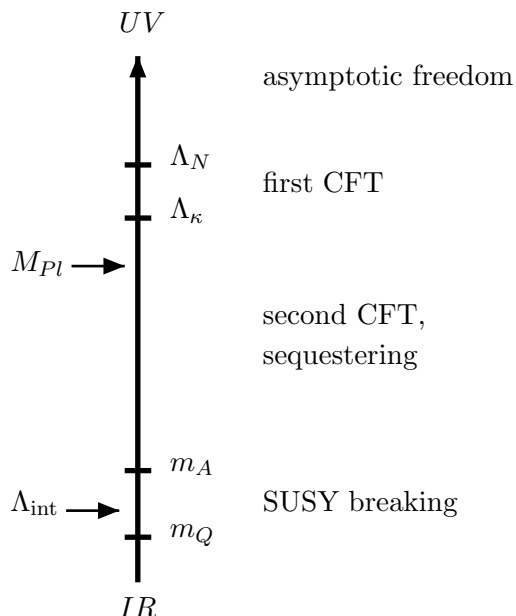


Figure 2: Scales and dynamics of our model. Λ_N and Λ_κ , respectively, are the scales at which the hidden sector gauge coupling and Yukawa coupling κ become strong and the theory transitions to a conformal fixed point. m_A and m_Q are masses which explicitly break conformal symmetry and trigger SUSY breaking. M_{Pl} and Λ_{int} are the Planck and intermediate scales.

coupling to be weak so that we can ignore its effects on the dynamics. Our superpotential is

$$W = m_Q Q\bar{Q} + \frac{m_A}{2} A^2 + \frac{\kappa}{3} A^3 \tag{3.2}$$

where the scales are arranged as shown in figure 2, $m_Q < m_A \ll \Lambda_\kappa \lesssim \Lambda_N$. This model was studied extensively in the context of supersymmetric duality in [27–29].

We will tell the story of the RG flow beginning above Λ_N , neglecting gravity, as a simplifying abstraction. Of course, in reality, (effective) field theory starts below M_{Pl} , and $\Lambda_N, \Lambda_\kappa$ are only apparent scales, mere theoretical crutches. Above Λ_N , the theory is weakly coupled and asymptotically free in the UV. At the scale Λ_N the $SU(N)$ gauge coupling becomes strong and drives the theory towards a strongly coupled fixed point. At this fixed point the operator κA^3 is relevant. Therefore κ grows quickly and near Λ_κ the theory approaches a new infrared fixed point at which both the $SU(N)$ gauge coupling and κ are strong. Scaling dimensions near this fixed point are quite different from the UV dimensions. For chiral primary operators they can be determined from superconformal R-charges. For example, $\text{dim}(Q\bar{Q}) = 3 - 2N/F$. The dynamics remains governed by this fixed point until conformal symmetry is broken by the mass of the adjoint field A (determined by m_A) and A decouples from the infrared theory.²

²The introduction of the hierarchy $m_X \ll \Lambda$ “by hand” may seem inelegant, but the small mass scales m_X can naturally arise from exponentially small non-perturbative effects. A concrete example is gaugino condensation due to a pure super-Yang-Mills sector with Planck suppressed couplings in the superpotential

Just below m_A the theory is SUSY QCD with a very strong gauge coupling and too few flavors to remain conformal. Instead, $SU(N)$ charges are confined and the theory flows to a free fixed point which is best described in terms of dual degrees of freedom. This dual has an $SU(F - N)$ gauge group and the particle content

	$SU(F - N)$	$SU(F)$	$SU(F)$
q	\square	$\bar{\square}$	1
\bar{q}	$\bar{\square}$	1	\square
M	1	\square	$\bar{\square}$

(3.3)

Here M is the operator map of the composite $SU(N)$ gauge invariant $Q\bar{Q}$ and q and \bar{q} are dual quarks. The dual has the superpotential

$$W = m^2 Tr M + Mq\bar{q} \quad (3.4)$$

where the mass scale $m^2 \propto m_Q$ is determined by matching the scaling dimension of the operator $Q\bar{Q}$ in the UV to the dimension of $Q\bar{Q}$ at the CFT fixed points and then to the free field operator M in the IR. Using the flavor symmetries, holomorphy, and by considering various limits, one can show that this superpotential is exact.

Finally, at energies of order m , the theory breaks SUSY with a metastable ground state [19]. To see this one uses the fact that the theory is IR free so that the kinetic terms for M, q, \bar{q} are approximately canonical and perturbation theory can be used to determine the vacuum structure. Assuming small vacuum expectation values compared to the scale m_A , the contribution to the potential from the F-term of M is

$$V \sim \sum_{i,j} |m^2 \delta_{ij} + q_{i\alpha} \bar{q}_j^\alpha|^2 + \dots \quad (3.5)$$

where i, j are $SU(F)$ flavor indices and α is an $SU(F - N)$ color index. This potential is necessarily nonzero because δ_{ij} is a matrix of rank F whereas $q_{i\alpha} \bar{q}_j^\alpha$ is at most of rank $F - N$ (q is an $(F - N) \times F$ matrix). Thus SUSY is spontaneously broken, and as in any O’Raifeartaigh model, there is a classical flat direction. It corresponds to the scalar components of some of the diagonal elements of M and is lifted once quantum corrections are taken into account. The dominant effect comes from one loop wave function renormalization of M due to the superpotential interaction $Mq\bar{q}$. The sign of this correction is such that M is stabilized at the origin. As discussed in [19] the theory also has supersymmetric vacua with large expectation values for M . Thus the SUSY breaking vacuum is only metastable but its lifetime is exponentially long in the parameter m_A/m_Q .

A nice feature of the ISS SUSY breaking model is that it is robust against perturbations to the UV physics. We explained that no new superpotential is generated when integrating out the field A . But a simple symmetry argument in the effective theory below the mass of A shows that even if additional terms were generated by non-perturbative physics, they

$\int d^2\theta \mathcal{W}_\alpha^2 (1 + \frac{X^2}{M_{Pl}^2}) \rightarrow \int d^2\theta \Lambda_{SYM}^3 (1 + \frac{X^2}{M_{Pl}^2})$ yielding a small mass term for X with negligibly small $\mathcal{O}(X^4/M_{Pl}^4)$ corrections.

would not destabilize SUSY breaking. The argument relies on the $SU(F) \times SU(F)$ flavor symmetry of the effective theory being only softly broken. The dynamically generated superpotential must be a function of the $SU(F) \times SU(F)$ invariants $m^2 M$, $q\bar{q}M$, and $\det(M)$ or $\det(q\bar{q})$. Here m^2 is the properly normalized spurion of the IR which transforms like m_Q of the UV. Furthermore, the superpotential must be regular in the IR fields M , q , \bar{q} , and therefore the most relevant terms which are not already included in (3.4) are $(m^2 M)^2/m_A$, $(Mq\bar{q})^2/m_A^3$ or $\det(M)/m_A^{F-3}$. They are too small to destabilize the vacuum for $F > 3$. This feature makes the ISS model very attractive for our purposes. The only significant constraint on UV modifications of the model is that the $SU(F) \times SU(F)$ flavor symmetry is not strongly broken in the conformal regime, as would happen if we were to introduce a $\bar{Q}AQ$ superpotential.

In summary, we find that between Λ_κ and m_A the theory is approximately conformal. Below m_A the adjoint A decouples and we are left with a strongly coupled non-conformal theory. In terms of the weakly coupled dual variables, the IR dynamics is easy to understand; SUSY is broken by the O’Raifeartaigh mechanism and a non-vanishing F-term for some diagonal components of M is generated.

We end this section by briefly outlining sequestering in our model, accounting for the fate of all operators of the form of eq. (1.1), with fuller derivation of sequestering given in the following sections. The only $SU(N) \times SU(F)$ gauge-invariant operators of the form of eq. (1.1) are those corresponding to hidden bilinears $X^\dagger X = (\bar{Q}^\dagger \bar{Q} \pm Q^\dagger Q), A^\dagger A$. Their dominant contribution to visible scalar masses can be determined by replacing the visible superfields Φ by their scalar components ϕ and treating these fields as spacetime constants, so that eq. (1.1) appears as a small ϕ -dependent coupling constant multiplying $\int d^4\theta X^\dagger X$. For the case $X^\dagger X = \bar{Q}^\dagger \bar{Q} - Q^\dagger Q$ this coupling can be field redefined away and therefore has no physical effect. For $X^\dagger X = (\bar{Q}^\dagger \bar{Q} + Q^\dagger Q), A^\dagger A$ these ϕ -dependent couplings cannot be redefined away completely, because the redefinitions induce small ϕ -dependent shifts of the $SU(N)$ gauge coupling (due to the Konishi anomaly [26]) and the A^3 Yukawa coupling away from their strong fixed-point values. The fact that these shifts are (technically) irrelevant physical couplings at the IR-attractive fixed point implies that the equivalent current operators, $\int d^4\theta(\bar{Q}^\dagger \bar{Q} + Q^\dagger Q), \int d^4\theta A^\dagger A$, are irrelevant too. Therefore ϕ -dependence is strongly suppressed in the IR. This is conformal sequestering.

4. Sequestering in detail

In the introduction we recounted a general plan for conformal sequestering, based on the hidden sector dynamics spending a large hierarchy of energies in the vicinity of a strongly IR-attractive fixed point prior to SUSY breaking. However, there are three subtleties one encounters in putting this plan to work:

- (i) The generic existence of relevant *superpotential* couplings in what are otherwise IR-attractive fixed points.
- (ii) The generic existence of marginal operators associated to global symmetries at fixed points.

(iii) We have thus-far neglected to properly integrate out (quadratic) fluctuations in the visible F_Φ -terms, rather than simply setting F_Φ to its vanishing VEV.

In this section, we will first discuss issues (i) and (ii), while continuing to make the “mistake” pointed out in (iii). We will finally return to a proper treatment of issue (iii). It is important to work through all these issues in order to properly understand and check how the model of section 3 accomplishes the tasks of conformal sequestering followed by SUSY breaking. The model also illustrates all the general issues in a relatively simple setting.

4.1 Relevant hidden superpotential couplings

The first subtlety is the fact that typical “IR-attractive” fixed points do indeed possess relevant perturbations, such as mass terms for some of the matter fields. A fixed point is usually deemed IR-attractive if such repulsive terms can be forbidden by symmetries, as is the case for any relevant superpotential couplings, including supersymmetric masses. However, in the present context we cannot simply impose such symmetries, because we in fact need small mass terms to ultimately drive the dynamics away from the fixed point towards SUSY breaking dynamics. According to our introduction we should then worry about terms in the Planck scale Lagrangian of the form,

$$\Delta\mathcal{L}_{\text{mixed}} \sim \phi^\dagger \phi \int d^2\theta X^n + \text{h.c.}, \tag{4.1}$$

where X^n represents some relevant hidden superpotential coupling. Fortunately though, such a term cannot arise from a full superspace invariant Lagrangian in terms of Φ and X (where we only keep the lowest component ϕ of the visible superfield Φ).

However, one can write unsequestered terms involving hidden superpotential couplings starting from mixed superpotentials,

$$\begin{aligned} \Delta\mathcal{L}_{\text{mixed}}(M) &\sim \int d^2\theta \frac{\Phi^k X^n}{M_{Pl}^{n+k-3}} + \text{h.c.} \\ &= \frac{\phi^k}{M_{Pl}^{n+k-3}} \int d^2\theta X^n + \text{h.c.} + \dots \end{aligned} \tag{4.2}$$

In the IR the strong hidden scaling results in

$$\Delta\mathcal{L}_{\text{mixed}}(\mu) \sim \left(\frac{\mu}{M_{Pl}}\right)^\gamma \frac{\phi^k}{M_{Pl}^{n+k-3}} \int d^2\theta X^n + \text{h.c.} + \dots \tag{4.3}$$

where γ is the anomalous dimension of X^n . Upon SUSY breaking this results in visible sector A-terms. This issue can be avoided by simply making the standard plausible assumption that the superpotential at the Planck scale is sequestered without a protective symmetry,

$$W(\Phi, X) = W_{\text{vis}}(\Phi) + W_{\text{hid}}(X), \tag{4.4}$$

but protected by the non-renormalization theorem.

Let us also consider the worst-case scenario in which the mixed superpotentials are indeed present and estimate their visible effects. Neglecting the thorny issue of the μ and $B\mu$ terms of the MSSM, let us focus on cubic visible gauge invariants, $k = 3$, and the resulting A-terms,

$$A_{\text{vis}} \sim \Lambda_{\text{int}} \left(\frac{\Lambda_{\text{int}}}{M_{Pl}} \right)^{n+\gamma}, \tag{4.5}$$

corresponding to a suppression of $(\frac{\Lambda_{\text{int}}}{M_{Pl}})^{n+\gamma-1}$ over directly-mediated visible soft terms. This estimate is obtained by assuming that the fixed point behavior operates until the theory is not far above Λ_{int} , and that this scale then sets all hidden sector VEVs. Note that $n + \gamma$ is the scaling dimension of X^n at the fixed point, which is bounded by unitarity to be ≥ 1 , corresponding to suppression over direct-mediation as long as the composite, X^n , is interacting.

The R-symmetry of the superconformal algebra at the fixed point determines the γ of chiral operators. In our model, there is a unique non-anomalous R-symmetry [27] which shows that the most relevant chiral operator is $\bar{Q}Q$, with $\gamma = 1 - 2N/F$, corresponding to suppression of A_{vis} relative to direct mediation of $(\Lambda_{\text{int}}/M_{Pl})^{2-2N/F}$. The danger is that this suppression disappears at the edge of the desired range, for $F \approx N$ where $M = Q\bar{Q}$ becomes a free field. Suppression of $(\frac{\Lambda_{\text{int}}}{M_{Pl}})^{1/2}$ is enough for A terms and therefore requires $F \gtrsim 4N/3$. Of course, the real lesson here is that we should stay away from parameter choices where parts of the CFT are nearly free fields, such as $\bar{Q}Q$ for $F \approx N$, since strong coupling is the key to sequestering.

To conclude this subsection, the relevance of possible hidden superpotential couplings does not destroy the plot of conformal sequestering, either because the Planck scale theory gives us a sequestered superpotential, or by ensuring that the entire hidden dynamics is fully and strongly interacting at the fixed point.

4.2 Marginal flavor-symmetry “currents”

The second subtlety is that at typical fixed points there are a set of global symmetries. These symmetries play an important role in the discovery of known strongly-coupled fixed points. The associated conserved Noether currents, J_μ^a , (except for R-symmetries) are contained in supermultiplets of the form $X^\dagger T^a X$, where T^a are matrices representing the symmetry generators acting on the X . Consequently, the standard vanishing of anomalous dimension for symmetry currents translates by SUSY to the vanishing of anomalous dimension for the entire supermultiplets $X^\dagger T^a X$. That is, there is a bilinear $X^\dagger T^a X$ (with hidden gauge multiplets implicit as needed for gauge invariance) of dimension exactly 2 for every global symmetry at a fixed point. Therefore, terms in the Planck-scale Lagrangian of the form

$$\begin{aligned} \Delta\mathcal{L}_{\text{mixed}} &= \frac{1}{M_{Pl}^2} \int d^4\theta \Phi^\dagger \Phi X^\dagger T^a X \\ &\rightarrow \frac{\phi^\dagger \phi}{M_{Pl}^2} \int d^4\theta X^\dagger T^a X + \dots, \end{aligned} \tag{4.6}$$

will not be conformally sequestered by the strong hidden fixed-point dynamics, indeed there is no running from this source at all!

We will use our hidden sector model to explain this loop-hole in our introductory argument guaranteeing sequestering, and to illustrate that while it poses the central danger in conformal sequestering, it is not fatal. To identify the global symmetries of the strong conformal dynamics let us formally shut off all perturbations in the hidden sector, namely the weak $SU(F)$ gauge theory and the mass terms, $\alpha_{SU(F)}, m_A, m_Q \rightarrow 0$. The exact (non-R) symmetries are then those familiar from SQCD, namely $SU(F)_Q \times SU(F)_{\bar{Q}} \times U(1)_{\text{baryon}}$, with associated current multiplets $Q^\dagger T^a Q, \bar{Q}^\dagger T^a \bar{Q}, Q^\dagger Q - \bar{Q}^\dagger \bar{Q}$, where the T^a span all traceless hermitian $F \times F$ matrices. These are dangerous because they have scaling (and canonical) dimension 2, corresponding to exactly marginal Kahler operators. The only other two $SU(N)$ gauge-invariant hidden-matter bilinears one can write are $Q^\dagger Q + \bar{Q}^\dagger \bar{Q}$ and $A^\dagger A$. But these bilinears (or any linear combinations) do not correspond to symmetries because of the strong axial anomaly of the $SU(N)$ gauge theory and strong explicit breaking by the fixed point A^3 Yukawa coupling. Since we are at a strong IR attractive fixed-point the corresponding Kahler operators $\int d^4\theta Q^\dagger Q + \bar{Q}^\dagger \bar{Q}, \int d^4\theta A^\dagger A$ are $\mathcal{O}(1)$ irrelevant and pose no danger.

To understand what makes the conserved current bilinears special let us perturb the fixed point Lagrangian infinitesimally by the associated Kahler terms. We can do this by starting in the asymptotic UV (for this theoretical exercise, we are shutting off gravity and the visible sector) with

$$\begin{aligned} \mathcal{L}_{UV} = & \int d^4\theta Q^\dagger Q + \bar{Q}^\dagger \bar{Q} + A^\dagger A + \int d^2\theta \tau \mathcal{W}^2 + \kappa A^3 + \text{h.c.} \\ & + \int d^4\theta \epsilon_a Q^\dagger T^a Q + \bar{\epsilon}_a \bar{Q}^\dagger T^a \bar{Q} + \epsilon(Q^\dagger Q - \bar{Q}^\dagger \bar{Q}), \end{aligned} \quad (4.7)$$

where the ϵ 's are infinitesimal. By the non-renormalization of the conserved currents this flows in the far IR to

$$\mathcal{L}_{IR} = \mathcal{L}_{\text{fixed-point}} + \int d^4\theta \epsilon_a Q^\dagger T^a Q + \bar{\epsilon}_a \bar{Q}^\dagger T^a \bar{Q} + \epsilon(Q^\dagger Q - \bar{Q}^\dagger \bar{Q}), \quad (4.8)$$

which is what we want to study. However, note that in the UV the ϵ terms are merely deviations from canonical normalization for the matter fields. We can return to canonical normalization by the field redefinitions,

$$\begin{aligned} Q & \rightarrow \left(I - \frac{\epsilon}{2} - \frac{\epsilon_a T^a}{2}\right) Q \\ \bar{Q} & \rightarrow \left(I + \frac{\epsilon}{2} - \frac{\bar{\epsilon}_a T^a}{2}\right) \bar{Q}. \end{aligned} \quad (4.9)$$

These field transformations are related by SUSY to the infinitesimal symmetry transformations obtained by rotation of the ϵ 's in the complex plane,

$$\begin{aligned} Q & \rightarrow \left(I - \frac{i\epsilon}{2} - \frac{i\epsilon_a T^a}{2}\right) Q \\ \bar{Q} & \rightarrow \left(I + \frac{i\epsilon}{2} - \frac{i\bar{\epsilon}_a T^a}{2}\right) \bar{Q}, \end{aligned} \quad (4.10)$$

and as such, neither transformation is anomalous. It is straightforward to see that this transformation leaves the superpotential invariant.³ Note that one cannot similarly transform away infinitesimal couplings of the form $Q^\dagger Q + \bar{Q}^\dagger \bar{Q}$ and $A^\dagger A$, because the transformations are either anomalous or broken by the strong superpotential coupling A^3 .

We thereby conclude that at the fixed point, the small change of wave-function normalization corresponding to the exact symmetry currents is physically irrelevant. That is, even though the local operator $\int d^4\theta X^\dagger T^a X(x)$ is a physical marginal operator at the fixed point, the associated Lagrangian term (zero momentum projection) $\int d^4x \int d^4\theta X^\dagger T^a X(x)$ is not physical, and therefore does not represent a marginal coupling of the fixed point (contradicting its being “IR attractive”).

4.3 Safe and unsafe currents

Of course, we are really interested in mixed couplings like eq. (4.6), which are certainly physical since ϕ is in general a function of x . However, for the purpose of determining the visible masses alone we can treat $\phi^\dagger \phi$ as constant in spacetime. Thus, we can think of the various $\phi^\dagger \phi$ terms in eq. (4.6) as the ϵ 's of eq. (4.7). If the hidden sector were only given by the fixed point dynamics, this would imply that the constant $\phi^\dagger \phi$ -dependence multiplying the symmetry currents can be completely transformed away as above, and no visible masses will result. However, the hidden sector also contains the perturbing couplings $\alpha_{\text{SU}(F)}, m_A, m_Q$. We begin by turning back on the mass terms, m_A, m_Q , but leaving $\text{SU}(F)$ still ungauged,

$$\begin{aligned} \mathcal{L}_{UV} = & \int d^4\theta Q^\dagger Q + \bar{Q}^\dagger \bar{Q} + A^\dagger A + \int d^2\theta \tau \mathcal{W}^2 + \kappa A^3 + m_A A^2 + m_Q \bar{Q} Q + \text{h.c.} \\ & + \int d^4\theta \epsilon_a Q^\dagger T^a Q + \bar{\epsilon}_a \bar{Q}^\dagger T^a \bar{Q} + \epsilon(Q^\dagger Q - \bar{Q}^\dagger \bar{Q}), \end{aligned} \tag{4.11}$$

where the ϵ 's represent $\phi^\dagger \phi$ constant visible bilinears. Now performing the transformation of eq. (4.9) leads to

$$\begin{aligned} \mathcal{L}_{UV} = & \int d^4\theta Q^\dagger Q + \bar{Q}^\dagger \bar{Q} + A^\dagger A \\ & + \int d^2\theta \tau \mathcal{W}^2 + \kappa A^3 + m_A A^2 + m_Q \bar{Q} \left(I - \frac{\epsilon_a + \bar{\epsilon}_a}{2} T^a \right) Q + \text{h.c.} \end{aligned} \tag{4.12}$$

The visible terms are not transformed away, but rather multiply m_Q . Since the SUSY breaking vacuum energy of the isolated hidden sector is $V_0 \propto |m_Q|^2$, in the presence of the visible sector perturbations, SUSY breaking leads to a potential

$$V_{\text{eff}} = V_0 \left(1 + \mathcal{O}\left(\frac{\phi^\dagger \phi}{M_{Pl}^2}\right) \right), \tag{4.13}$$

that is, unacceptable unsequestered visible scalar masses which dominate over AMSB. We will deal with this in the next subsection.

³In general the holomorphicity of superpotentials means that if they are invariant under symmetry group transformations with real parameters, ϵ , they are automatically invariant under the corresponding complexified group transformations for complex ϵ .

Note that the bilinear associated to $U(1)_{\text{baryon}}$ never had a chance to contribute to visible scalar masses because this symmetry is a symmetry of both the fixed point *and* the perturbations needed for SUSY breaking. As can be seen in eq. (4.12), the bilinear which couples to the baryon number current is totally transformed away by eq. (4.9).

4.4 Gauging flavor to suppress harmful non-abelian currents

Let us turn back on our weak gauging of the vectorial $SU(F)$ symmetry. Since the current bilinears $Q^\dagger T^a Q$, $\bar{Q}^\dagger T^a \bar{Q}$ are adjoints of this weak gauge group, $SU(F)$ gauge-invariance forbids the mixed couplings of the form $\epsilon_a, \bar{\epsilon}_a$. Consequently, our model is indeed fully sequestered.

4.5 Strong superpotential coupling to suppress harmful abelian current

Given our hidden field content, there is an alternative strong IR fixed point we might have thought to employ, namely the one arrived at by omitting the strong A^3 Yukawa coupling ($\kappa = 0$). However, this would have yielded one more (non-anomalous) $U(1)$ fixed point symmetry, corresponding to the bilinear $Q^\dagger Q + \bar{Q}^\dagger \bar{Q} - \frac{F}{N} A^\dagger A$. It is straightforward to check that visible couplings to this bilinear can be transformed away, but would result in $\phi^\dagger \phi$ dependence multiplying both m_A and m_Q , again resulting in a breakdown of sequestering. This $U(1)$ bilinear coupling is impossible to forbid by any weak gauging as in the previous subsection, because $U(1)$ currents are always singlets of any symmetry group, even the $U(1)$ itself. The strong A^3 Yukawa coupling was therefore crucial in having the resultant fixed point strongly break the $U(1)$ symmetry, turning the associated bilinear into an $\mathcal{O}(1)$ irrelevant coupling.

4.6 Emergent symmetries?

It should be stressed that the symmetries that pose a threat to conformal sequestering are the global symmetries of the strong hidden dynamics at the fixed point. The strong fixed point is often arrived at theoretically by RG flowing from a weakly coupled theory in the UV. Necessarily, any global symmetry of the weakly coupled parent theory is also a symmetry of the strong IR fixed point. In our model, this symmetry is $SU(F)_Q \times SU(F)_{\bar{Q}} \times U(1)_{\text{baryon}}$, discussed above. However, in addition the strong IR fixed point may have “emergent” or accidental symmetries not present in the UV parent theory. While such symmetries are equally dangerous to conformal sequestering, their existence is clearly much more difficult to ascertain because the weakly coupled UV parent cannot be consulted. One approach is to plausibly conjecture that such emergent symmetries are simply absent at the IR fixed point. But when there are dual descriptions of such fixed points, these often offer a powerful check of such a conjecture. This is the case in our model.

The IR fixed point we employ has a dual description [27–29] in terms of an $SU(2N-F)$ gauge theory with a superpotential schematically of the form,

$$W_{\text{dual}} \sim M \bar{q} a q + N \bar{q} q + a^3, \quad (4.14)$$

where \bar{q}, q are F flavors of dual quarks, a is a dual adjoint field and M, N are gauge-singlet flavor-bifundamental meson fields. This dual parent theory has the same global

symmetries as the original parent, namely $SU(F)_Q \times SU(F)_{\bar{Q}} \times U(1)_{\text{baryon}}$. However, if $M\bar{q}aq$ was irrelevant in the IR, as suggested by canonical power-counting, then M would decouple from the dynamics and become a free field. In that case there would be an additional $U(F^2)$ symmetry at the fixed point which transforms the decoupled fields M freely among themselves. The true relevance of the $M\bar{q}aq$ superpotential coupling can be determined from the superconformal R-charges and imposing the unitarity constraint that any gauge invariant chiral primary field must have scaling dimensions greater or equal to one. One finds that for $F \leq N$ this coupling must be irrelevant because otherwise M 's dimensions would be less than one. Thus the dual is allowing us to see the free field M and the associated emergent symmetries at the fixed point explicitly. Fortunately however, the SUSY breaking mechanism of ref. [19] operates for $F > N$, where one finds that the $M\bar{q}aq$ coupling is strong at the IR fixed point and as important as any of the other superpotential couplings. Thus, in our model, no new symmetries emerge in the dual description, and the conjecture that the only symmetries are $SU(F)_Q \times SU(F)_{\bar{Q}} \times U(1)_{\text{baryon}}$ is strengthened.

An expert reader may have noticed that it was not really necessary to consult the dual theory to determine if the gauge invariant $M = Q\bar{Q}$ becomes a free field. Already in the electric theory one can find the scaling dimension of M from its superconformal R charge and see that its dimension approaches 1 as F approaches N from above, suggesting that M is a free field for $F \leq N$.

However there is another worry regarding emergent symmetries which only becomes apparent in the dual. The worry is that the $U(1)_A$ symmetry which we engineered to be broken explicitly by the A^3 term might re-emerge at the fixed point. How might this happen? Consider adding the A^3 term with small coefficient κ to the CFT with vanishing superpotential. At the $\kappa = 0$ fixed point the dimension of A^3 can be determined using R-charges and a-maximization [30]. One finds that the operator A^3 is relevant, driving the theory away from $\kappa = 0$ towards large values of κ . This is good because it shows that our desired theory (with $U(1)_A$ broken explicitly) is not unstable to flowing back to the $\kappa = 0$ fixed point, and we now have supporting evidence that the fixed point with κ turned on strongly really does exist. What about the dual? In the dual, the $U(1)_A$ symmetry is only broken by the term $\tilde{\kappa}a^3$. Therefore, just as with κA^3 , we must worry that our desired fixed point with $U(1)_A$ broken might not actually exist because it is unstable to flowing towards the fixed point where $\tilde{\kappa} = 0$. Happily, the superconformal R-charges again determine that for small $\tilde{\kappa}$ the interaction a^3 is relevant, and therefore the theory also flows away from this bad fixed point at which $U(1)_A$ is restored. Altogether we have strong evidence that the desired fixed point with no emergent symmetries exists and that it is stable.⁴

⁴For an example where the dual provides evidence that a symmetry similar to $U(1)_A$ emerges at a fixed point consider a slightly different version of our model. Start with the ISS model and make it conformal not by adding a massive adjoint A and its superpotential A^3 but instead by adding $N/2$ extra massive flavors P, \bar{P} with the superpotential $(P\bar{P})^2$. As in the case of the adjoint, the superpotential was designed to break a dangerous axial $U(1)_A$, and one can easily see from the R symmetry that $(P\bar{P})^2$ is relevant (when added with small coefficient) in the whole range of interest $N < F < 3N/2$. The R symmetry also shows that the meson $M = Q\bar{Q}$ becomes free for $F \leq 9N/8$. Therefore it seems that the model should sequester as long as $9N/8 < F < 3N/2$. However, in the Seiberg dual we discover a potential problem. Like in the original variables $U(1)_A$ symmetry is broken by the superpotential $\tilde{\kappa}(p\bar{p})^2$. However this time we find that $(p\bar{p})^2$ is

4.7 Integrating out visible F_Φ -terms

Let us now take up point (iii) raised at the beginning of this section, namely that until now we have been ignoring visible auxiliary fields F_Φ within mixed visible-hidden terms,

$$\begin{aligned}\Delta\mathcal{L}_{\text{mixed}}(M) &\sim \frac{1}{M_{Pl}^n} \int d^4\theta \Phi^\dagger \Phi \mathcal{O}_{\text{hid}} \\ \Delta\mathcal{L}_{\text{mixed}}(\mu) &\sim \frac{\mu^\gamma}{M_{Pl}^{n+\gamma}} \int d^4\theta \Phi^\dagger \Phi \mathcal{O}_{\text{hid}},\end{aligned}\tag{4.15}$$

where here \mathcal{O}_{hid} denotes a (composite) hidden superfield with canonical dimension n and fixed-point scaling dimension $n+\gamma$. Assuming the fixed-point scaling holds down to energies modestly larger than Λ_{int} , and integrating out the visible auxiliary field, results in visible scalar mass contributions,

$$m_\phi^2 \sim \frac{\Lambda_{\text{int}}^4}{M_{Pl}^2} \left(\frac{\Lambda_{\text{int}}}{M_{Pl}}\right)^{2(n+\gamma-1)}.\tag{4.16}$$

This is suppressed compared to direct mediation contributions, by $(\Lambda_{\text{int}}/M_{Pl})^{2(n+\gamma-1)}$. By the unitarity constraints on conformal scaling dimensions, indeed $n + \gamma > 1$, as required for suppression.

In general, we are unable to compute the strong anomalous dimensions γ , and we rely on the fact that they are expected to be $\mathcal{O}(1)$ to provide sufficient suppression of this class of visible SUSY breaking in order for AMSB to dominate. There is however a special case when the hidden operator is chiral, $\mathcal{O}_{\text{hid}} \sim X^n + \text{h.c.}$, for which the superconformal R-charge determines γ . Actually, in this instance eq. (4.15) can be field redefined away via

$$\Phi \rightarrow \Phi \left(1 - \frac{X^n}{M_{Pl}^n}\right),\tag{4.17}$$

at the cost of leading to hidden contamination of the visible superpotential,

$$W_{\text{vis}}(\Phi) \rightarrow W_{\text{vis}}\left(\Phi \left(1 - \frac{X^n}{M_{Pl}^n}\right)\right).\tag{4.18}$$

But the case of such visible-hidden mixed superpotentials has already been covered above.

5. Discussion and generalizations

In this section we collect and generalize our results and discuss their robustness. When analyzing candidates for models of hidden sectors with dynamical SUSY breaking and conformal sequestering one must perform a few consistency checks.

actually irrelevant in the theory with $\tilde{\kappa}$ turned off for $F < 1.24991N$. In this regime of flavors the theory actually flows back towards $\tilde{\kappa} = 0$ and $U(1)_A$ re-emerges. This makes the desired fixed point at which $U(1)_A$ is strongly broken highly suspicious because of the existence of an alternative attractive fixed point nearby. We conclude that this modified model appears to sequester safely only for the relatively narrow range of flavors $1.24991N < F < 1.5N$.

Consistency checks for dynamical SUSY breaking. When integrating out heavy states at the scale of conformal symmetry breaking one must check that dynamical superpotentials are not generated which would otherwise ruin the IR SUSY breaking model. We distinguish two cases, depending on whether conformal symmetry breaking is explicit or spontaneous.

In the case of spontaneous breaking of conformal symmetry, there is a Nambu-Goldstone boson, the “dilaton”. The flatness of its potential implies that no dynamical superpotential can be generated. Instead explicit breaking by irrelevant operators must be used in order to generate a weak stabilizing dilaton potential [11, 12].

The other case, explicit conformal symmetry breaking by a relevant operator, as in this paper, is conceptually simpler. However, the relevant operator necessarily strongly breaks the superconformal R-symmetry, which is dangerous because R-symmetries are usually what prevent a dynamical superpotential from being generated. But in our model, the softly broken $SU(F) \times SU(F)$ flavor symmetries of the low energy effective theory (the ISS model) together with holomorphy forbid any new superpotential couplings which are relevant between the scales of conformal symmetry breaking and SUSY breaking. Dynamically generated superpotential couplings which are irrelevant between these scales do not destabilize SUSY breaking because in our model their effect on SUSY breaking can be made arbitrarily small, since the hierarchy between conformal breaking and SUSY breaking is a free parameter of the model.

Thus our SUSY breaking model is robust. In fact, it is easy to construct other UV extensions of the ISS model which also exhibit conformal sequestering. The only constraint from SUSY breaking is that the $SU(F) \times SU(F)$ flavor symmetry be preserved in the conformal regime.

Consistency checks for conformal sequestering Here one needs to make sure that all possible couplings between the hidden and visible sectors are sequestered. As explained in this paper, the most dangerous couplings are relevant and marginal operators of the CFT coupled to MSSM bilinears. The couplings to relevant operators (which are gauge and superpotential couplings and therefore chiral) can be forbidden by symmetries and are not a problem. More care is required in studying possible marginal operators. In particular, for every global symmetry of a CFT there exists a supermultiplet $X^\dagger T^a X$ which contains the symmetry current and which is exactly marginal. Therefore operators of the form

$$\frac{\Phi^\dagger \Phi}{M_{Pl}^2} X^\dagger T^a X \tag{5.1}$$

are not renormalized by the conformal dynamics and do not sequester. Sometimes these dangerous symmetries at a conformal fixed point are difficult to spot because they are “emergent” or accidental as discussed in subsection 4.6. Yet often they become apparent in dual descriptions of the CFT.

To understand whether an operator like eq. (5.1) is problematic we distinguish four cases:

1. *Exact symmetries of the entire hidden sector.* In this case, field transformations can be used to remove the couplings of the hidden currents to the MSSM bilinear, as discussed in section 4.2 and illustrated by (4.9). From this we conclude that even though the operator (5.1) does not scale to zero it does not give rise to scalar masses. This implies the following:

If the action of a model of spontaneous SUSY breaking has an exactly preserved global symmetry then the D component of the corresponding supercurrent has a vanishing expectation value. i.e.

$$\langle X^\dagger T^a e^V X \Big|_D \rangle = \langle F^\dagger T^a F \rangle + \langle x^\dagger T^a D x \rangle = 0 \quad (5.2)$$

where x and F are the scalar and F-components of X , and D is a D-term of hidden sector gauge fields. This is true even when the global symmetry is spontaneously broken.

2. *Non-abelian symmetries.* The coupling of non-Abelian currents to the visible sector can typically be forbidden by weakly gauging a subgroup of the non-Abelian symmetry group.
3. *Symmetries of the CFT dynamics which are broken by relevant operators in the superpotential.* In this case the operator (5.1) does not sequester because while it contains a conserved current of the CFT we cannot use the complexified symmetry transformations, such as eq. (4.9), to remove it because the corresponding symmetry is broken. In this case sequestering fails and such approximate symmetries must be avoided. In practice this means adding new interactions to the CFT which cause it to flow to a new CFT where the symmetry is strongly broken. These may be new strong gauge interactions or superpotential interactions like the κA^3 in our model. This leads to our next case below.
4. *“Symmetries” which are strongly broken by the CFT dynamics.* In this case the bilinear $X^\dagger T^a X$ does not correspond to a conserved current because its associated symmetry is strongly broken by gauge or superpotential couplings at the fixed point. Since we are considering conformal fixed points which are IR attractive the operator $X^\dagger T^a X$ must be irrelevant, and the coupling (5.1) scales to zero between the Planck scale and the SUSY breaking scale. At low energies it is suppressed by an additional $(\Lambda_{\text{int}}/M_{Pl})^\gamma$ where the incalculable anomalous dimension γ is expected to be of order 1.

In conclusion, we have presented a class of hidden sector models which exhibit conformal sequestering and break SUSY dynamically. Sequestering is important as it is a necessary ingredient in models where SUSY breaking is mediated at a high scale (such as anomaly-, high scale gauge or gaugino-, or graviton loop-mediation). We demonstrated that conformal sequestering occurs in renormalizable four-dimensional models without relying on assumptions about Planck scale physics or fortuitous discrete symmetries. The simplicity and robustness of our models leads us to believe that conformal sequestering is generic in the “landscape” of possible hidden sectors with dynamical SUSY breaking.

Acknowledgments

We thank Andy Cohen, Paddy Fox, Ken Intriligator, Ami Katz, David Kaplan, Yuri Shirman, Brian Wecht for useful discussions and the Aspen Center for Physics, the CERN Theory Group, the Technion, and the Galileo Galilei Institute for their hospitality while this research was nearing completion. M.S. is supported by the DOE OJI grant DE-FG02-91ER40676 and an Alfred P. Sloan Research Fellowship. R.S. is supported by NSF grant P420-D36-2043-4350.

A. SUSY QCD Sequestering

The Lagrangian for supersymmetric QCD (N colors, F flavors) in the holomorphic basis is

$$\mathcal{L}_{\text{hol}} = Z (Q_{\text{hol}}^\dagger Q_{\text{hol}} + \bar{Q}_{\text{hol}}^\dagger \bar{Q}_{\text{hol}}) \Big|_D + (\tau W_\alpha W^\alpha|_F + h.c.) \quad (\text{A.1})$$

where τ is the usual holomorphic coupling constant which runs at one loop

$$\tau(\mu) = \tau(M) + \frac{b}{8\pi^2} \log\left(\frac{\mu}{M}\right), \quad b = 3N - F \quad (\text{A.2})$$

and the wave function $Z(\mu)$ gets contributions from all orders in perturbation theory. In this basis, it is difficult to see how supersymmetric QCD could be conformal for any number of flavors except $F = 3N$ because τ is manifestly μ dependent. The resolution of this puzzle lies in the running of the wave function of the matter fields: Classically, the wave function factor Z for the matter fields is unphysical, it can be rescaled out of the Lagrangian by redefining fields $\hat{Q} = \sqrt{Z} Q_{\text{hol}}$, $\hat{\bar{Q}} = \sqrt{Z} \bar{Q}_{\text{hol}}$. Quantum mechanically this transformation is anomalous [26] and results in a shift of the coupling by $-\frac{F}{8\pi^2} \log Z$. Thus in the basis where the kinetic terms for the matter fields are canonical the running coupling is given by [31, 32]

$$\hat{\tau}(\mu) = \tau(M) + \frac{b}{8\pi^2} \log\left(\frac{\mu}{M}\right) - \frac{F}{8\pi^2} \log Z(\mu) \quad (\text{A.3})$$

which is scale independent when the μ dependence of Z is of the form

$$Z(\mu) = Z(M) \left(\frac{\mu}{M}\right)^{2\gamma_Q}, \quad 2\gamma_Q = \frac{b}{F} = \frac{3N - F}{F} \quad (\text{A.4})$$

The basis in which both Z and τ are running even though the theory has only one physical coupling is confusing when discussing conformal field theories. The two alternatives *i.* canonical kinetic terms and all running in $\hat{\tau}$ and *ii.* fixing the gauge coupling by moving the one-loop running of τ into the wave functions of the matter fields are more convenient. In the following we give explicit formulae for the running couplings in all three bases.

We start with the canonical basis in which the only running is in the gauge coupling.⁵ The beta function expanded near the fixed point $\hat{\tau} = \tau_*$ is

$$\beta = 0 + \beta'_* (\hat{\tau} - \tau_*) \quad (\text{A.5})$$

⁵Note that we use canonical kinetic terms for the matter fields but not for the gauge fields.

with the solution

$$\hat{\tau}(\mu) - \tau_* = \left(\frac{\mu}{M}\right)^{\beta'_*} (\hat{\tau}(M) - \tau_*). \quad (\text{A.6})$$

Thus the running Lagrangian in this basis is

$$\hat{\mathcal{L}} = (\hat{Q}^\dagger \hat{Q} + \hat{\bar{Q}}^\dagger \hat{\bar{Q}}) \Big|_D + \left(\left[\tau_* + \left(\frac{\mu}{M}\right)^{\beta'_*} (\hat{\tau}(M) - \tau_*) \right] W_\alpha W^\alpha \Big|_F + h.c. \right) \quad (\text{A.7})$$

We now switch to the basis which we find most useful to discuss conformal sequestering, the basis where all running takes place in the kinetic terms. The scaling of the kinetic term is easily obtained by doing the general field redefinitions $\hat{Q} = \sqrt{R}Q$, $\hat{\bar{Q}} = \sqrt{R}\bar{Q}$ under which the gauge coupling shifts by $\frac{F}{8\pi^2} \log(R)$ and solving for R such that the new gauge coupling is fixed at τ_* . The result is

$$\mathcal{L} = [R(M)]^{\left(\frac{\mu}{M}\right)^{\beta'_*}} (Q^\dagger Q + \bar{Q}^\dagger \bar{Q}) \Big|_D + (\tau_* W_\alpha W^\alpha \Big|_F + h.c.) \quad (\text{A.8})$$

Note that the wave function factor rapidly approaches 1 as $\mu \rightarrow 0$. For $R(M) \simeq 1$ it may be expanded to give

$$R(\mu) = 1 + \left(\frac{\mu}{M}\right)^{\beta'_*} (R(M) - 1) \quad (\text{A.9})$$

which is the formula we used earlier in (2.11).

For the purpose of conformal sequestering we are worried about operators of the form

$$c \frac{\Phi^\dagger \Phi}{M^2} (Q^\dagger Q + \bar{Q}^\dagger \bar{Q}) \Big|_D \quad (\text{A.10})$$

We may derive their scaling due to hidden sector interactions by considering constant (scalar) background values for the field Φ so that $c \frac{\Phi^\dagger \Phi}{M^2}$ becomes simply a number which contributes to the kinetic terms of Q and \bar{Q} i.e. it contributes a shift to $R(M)$. But we know that all Lagrangian-dependence on $R(M)$ is suppressed by a factor of $\left(\frac{\mu}{M}\right)^{\beta'_*}$, therefore our operator must be sequestered

$$\mathcal{O}(\mu) = \left(\frac{\mu}{M}\right)^{\beta'_*} c \frac{\Phi^\dagger \Phi}{M^2} (Q^\dagger Q + \bar{Q}^\dagger \bar{Q}) \Big|_D \quad (\text{A.11})$$

For completeness and to confuse you, we also give the running Lagrangian in the holomorphic basis which is obtained from eq. (A.8) by moving the one-loop running back into the gauge coupling with the transformation $Q = \left(\frac{\mu}{M}\right)^{\gamma_Q} Q_{\text{hol}}$. The Lagrangian is then [11]

$$\mathcal{L}_{\text{hol}} = [R(M)]^{\left(\frac{\mu}{M}\right)^{\beta'_*}} \left(\frac{\mu}{M}\right)^{2\gamma_Q} (Q_{\text{hol}}^\dagger Q_{\text{hol}} + \bar{Q}_{\text{hol}}^\dagger \bar{Q}_{\text{hol}}) \Big|_D \quad (\text{A.12})$$

$$+ \left(\left[\tau_* + \frac{b}{8\pi^2} \log\left(\frac{\mu}{M}\right) \right] W_\alpha W^\alpha \Big|_F + h.c. \right) \quad (\text{A.13})$$

Now it appears that there might be an extra suppression in conformal sequestering due to the $\left(\frac{\mu}{M}\right)^{2\gamma_Q}$. But this factor appears in front of the operator which couples visible and hidden sector as well as in front of the kinetic terms for the Q_{hol} 's. Thus when we canonically normalize hidden sector fields this factor drops out again. This makes it clear that the anomalous scaling dimensions at the fixed point γ_Q do not contribute to sequestering.

References

- [1] L. Randall and R. Sundrum, *Out of this world supersymmetry breaking*, *Nucl. Phys. B* **557** (1999) 79 [[hep-th/9810155](#)].
- [2] G.F. Giudice, M.A. Luty, H. Murayama and R. Rattazzi, *Gaugino mass without singlets*, *JHEP* **12** (1998) 027 [[hep-ph/9810442](#)].
- [3] D.E. Kaplan, G.D. Kribs and M. Schmaltz, *Supersymmetry breaking through transparent extra dimensions*, *Phys. Rev. D* **62** (2000) 035010 [[hep-ph/9911293](#)].
- [4] Z. Chacko, M.A. Luty, A.E. Nelson and E. Ponton, *Gaugino mediated supersymmetry breaking*, *JHEP* **01** (2000) 003 [[hep-ph/9911323](#)].
- [5] T. Gregoire, R. Rattazzi, C.A. Scrucca, A. Strumia and E. Trincherini, *Gravitational quantum corrections in warped supersymmetric brane worlds*, *Nucl. Phys. B* **720** (2005) 3 [[hep-th/0411216](#)].
- [6] T. Gregoire, R. Rattazzi and C.A. Scrucca, *D-type supersymmetry breaking and brane-to-brane gravity mediation*, *Phys. Lett. B* **624** (2005) 260 [[hep-ph/0505126](#)].
- [7] K. Choi, A. Falkowski, H.P. Nilles, M. Olechowski and S. Pokorski, *Stability of flux compactifications and the pattern of supersymmetry breaking*, *JHEP* **11** (2004) 076 [[hep-th/0411066](#)].
- [8] K. Choi, A. Falkowski, H.P. Nilles and M. Olechowski, *Soft supersymmetry breaking in KKL_T flux compactification*, *Nucl. Phys. B* **718** (2005) 113 [[hep-th/0503216](#)].
- [9] K. Choi, K.S. Jeong, T. Kobayashi and K.-i. Okumura, *Little SUSY hierarchy in mixed modulus-anomaly mediation*, *Phys. Lett. B* **633** (2006) 355 [[hep-ph/0508029](#)].
- [10] R. Kitano and Y. Nomura, *A solution to the supersymmetric fine-tuning problem within the MSSM*, *Phys. Lett. B* **631** (2005) 58 [[hep-ph/0509039](#)].
- [11] M.A. Luty and R. Sundrum, *Supersymmetry breaking and composite extra dimensions*, *Phys. Rev. D* **65** (2002) 066004 [[hep-th/0105137](#)].
- [12] M. Luty and R. Sundrum, *Anomaly mediated supersymmetry breaking in four dimensions, naturally*, *Phys. Rev. D* **67** (2003) 045007 [[hep-th/0111231](#)].
- [13] A.E. Nelson and M.J. Strassler, *Suppressing flavor anarchy*, *JHEP* **09** (2000) 030 [[hep-ph/0006251](#)].
- [14] A.E. Nelson and M.J. Strassler, *Exact results for supersymmetric renormalization and the supersymmetric flavor problem*, *JHEP* **07** (2002) 021 [[hep-ph/0104051](#)].
- [15] A. Anisimov, M. Dine, M. Graesser and S.D. Thomas, *Brane world SUSY breaking from string/m theory*, *JHEP* **03** (2002) 036 [[hep-th/0201256](#)].
- [16] S. Kachru, J. McGreevy and P. Svrcek, *Bounds on masses of bulk fields in string compactifications*, *JHEP* **04** (2006) 023 [[hep-th/0601111](#)].
- [17] M. Ibe, K.I. Izawa, Y. Nakayama, Y. Shinbara and T. Yanagida, *Conformally sequestered SUSY breaking in vector-like gauge theories*, *Phys. Rev. D* **73** (2006) 015004 [[hep-ph/0506023](#)].
- [18] M. Ibe, K.I. Izawa, Y. Nakayama, Y. Shinbara and T. Yanagida, *More on conformally sequestered SUSY breaking*, *Phys. Rev. D* **73** (2006) 035012 [[hep-ph/0509229](#)].

- [19] K. Intriligator, N. Seiberg and D. Shih, *Dynamical SUSY breaking in meta-stable vacua*, *JHEP* **04** (2006) 021 [[hep-th/0602239](#)].
- [20] R. Sundrum, *'Gaugomally' mediated SUSY breaking and conformal sequestering*, *Phys. Rev. D* **71** (2005) 085003 [[hep-th/0406012](#)].
- [21] T. Kobayashi and H. Terao, *Sfermion masses in Nelson-Strassler type of models: SUSY standard models coupled with SCFTs*, *Phys. Rev. D* **64** (2001) 075003 [[hep-ph/0103028](#)].
- [22] T. Kobayashi, H. Nakano and H. Terao, *Yukawa hierarchy transfer from superconformal sector and degenerate sfermion masses*, *Phys. Rev. D* **65** (2002) 015006 [[hep-ph/0107030](#)].
- [23] T. Kobayashi, H. Nakano, T. Noguchi and H. Terao, *Sfermion mass degeneracy, superconformal dynamics and supersymmetric grand unified theories*, *Phys. Rev. D* **66** (2002) 095011 [[hep-ph/0202023](#)].
- [24] M. Dine et al., *Visible effects of the hidden sector*, *Phys. Rev. D* **70** (2004) 045023 [[hep-ph/0405159](#)].
- [25] N. Arkani-Hamed, G.F. Giudice, M.A. Luty and R. Rattazzi, *Supersymmetry-breaking loops from analytic continuation into superspace*, *Phys. Rev. D* **58** (1998) 115005 [[hep-ph/9803290](#)].
- [26] K. Konishi, *Anomalous supersymmetry transformation of some composite operators in sqcd*, *Phys. Lett. B* **135** (1984) 439.
- [27] D. Kutasov, *A comment on duality in $N = 1$ supersymmetric nonAbelian gauge theories*, *Phys. Lett. B* **351** (1995) 230 [[hep-th/9503086](#)].
- [28] D. Kutasov and A. Schwimmer, *On duality in supersymmetric Yang-Mills theory*, *Phys. Lett. B* **354** (1995) 315 [[hep-th/9505004](#)].
- [29] D. Kutasov, A. Schwimmer and N. Seiberg, *Chiral rings, singularity theory and electric-magnetic duality*, *Nucl. Phys. B* **459** (1996) 455 [[hep-th/9510222](#)].
- [30] K. Intriligator and B. Wecht, *The exact superconformal R-symmetry maximizes a*, *Nucl. Phys. B* **667** (2003) 183 [[hep-th/0304128](#)].
- [31] V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Exact gell-mann-low function of supersymmetric Yang-Mills theories from instanton calculus*, *Nucl. Phys. B* **229** (1983) 381.
- [32] N. Arkani-Hamed and H. Murayama, *Holomorphy, rescaling anomalies and exact beta functions in supersymmetric gauge theories*, *JHEP* **06** (2000) 030 [[hep-th/9707133](#)].